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EXPLORING THE WAYS IN WHICH THE GOLDEN RATIO CAN BE DERIVED

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ABSTRACT—*This research paper gives a wholesome overview of the Golden ratio and its connection to the Fibonacci sequence. The Golden ratio is an omnipresent proportion that objectively gives beauty to things ranging from architecture and portraits to living things and makes them pleasing to the eye. As a result, this ever-present ratio has a variety of applications in fields such as art, astronomy and economics. This report is primarily concerned with giving an overview of the Golden ratio and discussing the various ways to determine it and investigate its wide-ranging applications.*

Keywords: *Mathematics; Applied Mathematics; The Golden Ratio; Fibonacci Sequence; Divine Proportion; 01-01; 03-01.*

QUOTES

"The Golden Ratio has inspired thinkers of all disciplines like no other number in the history of Mathematics"[1] – Mario Livio

"If it looks, smells, tastes, feels, or sounds nice then 'The Golden Ratio' is hidden in there somewhere"[2] – Anonymous

*"**The golden ratio** is a reminder of the relatedness of the created world to the perfection of its source and of its potential future evolution."[3] — **Robert Lawlor***

INTRODUCTION

The objectivity of beauty was brought to light by mathematician Leonardo Fibonacci in the 1200s while he was in the process of discovering the properties of the **Fibonacci sequence**. "The Fibonacci sequence is a series of numbers where each term is the sum of the two terms preceding it."[4] A very special property of the Fibonacci sequence is that if any two consecutive numbers in the sequence are taken and the larger number is divided by the smaller number, a ratio that is very close to the following number will be obtained: 1.61803398875. As the values of the two numbers gradually increase, their ratio will tend to the number above. This very special number is called the **Golden Ratio**, and will be the central focus of this research report.

[5]This very numerical value is considered to represent something called the 'golden proportion'. In other words, any particular thing, be it a piece of art or the human face, with features that have proportions close to 1:1.61803398875, is considered to be extremely beautiful.

Most mesmerising things are said to entail this golden ratio somewhere within them. The golden ratio can be seen in numerous pieces of art, ranging from the Great Pyramid of Giza to the Mona Lisa painting to Michelangelo's *The Creation of Adam*. It also plays a central role in the award winning movie "The Da Vinci Code".

[1] "Golden Ratio Quote by Mario Livio | Golden Ratio - Golden ... - Pinterest." https://www.pinterest.com/pin/208150814000567292/. Accessed 7 Jun. 2018.

[2] "Best goldenratio Quotes, Status, Shayari, Poetry & Thoughts | YourQuote." https://www.yourquote.in/tags/goldenratio/quotes. Accessed 7 Jun. 2018.

[3] "The Golden Ratio Quotes - Wise Famous Quotes." https://www.wisefamousquotes.com/quotes-about-the-golden-ratio/. Accessed 7 Jun. 2018.

[4] "Fibonacci Sequence - Math is Fun." https://www.mathsisfun.com/numbers/fibonacci-sequence.html. Accessed 8 Jul. 2018.

[5] Image from Google Images - https://www.google.com/search?q=mona+lisa&source=lnms&tbm=isch&sa=X&ved=0ahUKEwipvNfl7qXjAhUJQ48KHVeIA1IQ_AUIECgB&biw=1280&bih=578&dpr=1.5#imgrc=eRLI-JSNfkRaIM:

The Golden Ratio is not only seen in art and architecture. One can also see its prevalence in biology and the environment. The number of petals on a flower is always a term of the Fibonacci sequence and the proportions on an insects body is quite close to the Golden Ratio. The Golden Ratio can also be applied to stock markets and finance. Human expectations can be represented in a ratio that clearly approaches the Golden ratio. The shape of galaxies, especially the shape of our Milky Way galaxy, is quite close to the shape of the 'Golden Spiral'[6].

There are various methods that can be used to find the golden ratio, proving its omnipresence.

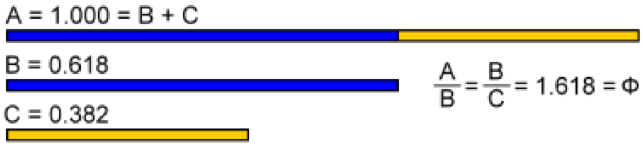
DIFFERENT WAYS TO FIND THE GOLDEN RATIO

The golden ratio has the value 1.61803398875, which can be rounded off to 1.618 to 3 decimal places. The Golden ratio is an irrational number and it is usually denoted by the symbol φ (phi). There are several ways to find the golden ratio, and this report will demonstrate some of them.

The golden ratio can be defined as the ratio of the length of the larger part of a line to the length of the smaller part of a line, given that this ratio is equal to the ratio of the length of the entire line to the length of the larger part. So, if the length of the entire line is a + b, the length of the larger part is a, and the length of the smaller part is b, then the following holds:

$$\frac{a+b}{a} = \frac{a}{b} \approx 1.618$$

Consider a rope of length 1 metre. There are various ways of cutting this rope into two pieces and obtaining a longer and shorter rope. However, there is only one particular point on the string at which the ratio of the larger rope to the shorter rope will be equal to the ratio of the entire rope to the larger rope. This point is at the 0.618 metre mark on the 1 metre rope.



[7]

For example, a rope of 30 centimetres can be cut into two pieces of lengths 18.54 cm and 11.46 cm (this is because 30 ÷ 1.618 = 18.54 and 30 ÷ (1.618 × 1.618) = 18.54 ÷ 1.618 = 11.46).

There are two interesting characteristics of the golden ratio:

1) The Golden ratio is the only number whose square is greater than the number itself by one. That is, $\varphi^2 = \varphi + 1 \approx 2.618$
2) It is also the only number whose reciprocal is less than the number itself by one. That is, $\frac{1}{\varphi} = \varphi - 1 \approx 0.618$

The above two relationships ($\varphi^2 = \varphi + 1$ and $\frac{1}{\varphi} = \varphi - 1$) can be simplified into one single quadratic equation:

$$\varphi^2 - \varphi - 1 = 0$$
$$\varphi = \frac{1 + \sqrt{1-(-4)}}{2}$$
$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

This is how the value of φ (the golden ratio) can be derives using its special relationships and the formula for finding the roots of a quadratic equation.

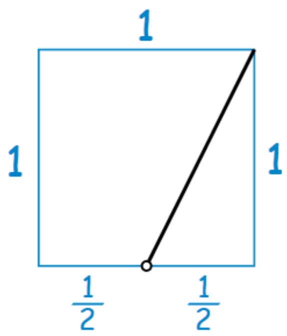
Another method that can be used to find the golden ratio is to use the Fibonacci sequence itself. If two consecutive numbers in the Fibonacci sequence are taken and the larger of the two numbers is divided by the smaller one, the result will be very close to the golden ratio. As the consecutive terms get larger, the ratio between the two numbers will get closer and closer to the golden ratio.

First number	Second number	Ratio
1	1	1.000
1	2	2.000
2	3	1.500
3	5	1.667
5	8	1.600
8	13	1.625
13	21	1.615
21	34	1.619
34	55	1.618
55	89	1.618
89	144	1.618
144	233	1.618
233	377	1.618

[6]A *golden spiral* is a logarithmic *spiral* whose growth factor is φ, the *golden* ratio. That is, a *golden spiral* gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. – ("Golden spiral - Wikipedia." https://en.wikipedia.org/wiki/Golden_spiral. Accessed 18 Oct. 2018.)

[7]Image from "Golden ratio properties, appearances and applications overview." 12 Jul. 2015, https://www.goldennumber.net/golden-ratio/. Accessed 29 Oct. 2018.

The ratio in the table above eventually converges to 1.618, which is the value of the golden ratio rounded off to 3 decimal places. (The ratio converges to the value of the golden ratio but never actually meets it. In the above table, the ratio is rounded off to 3 decimal places, and that's why it might be the same as the golden ratio rounded to 3 decimal places, but it will differ from it when we increase the decimal places).



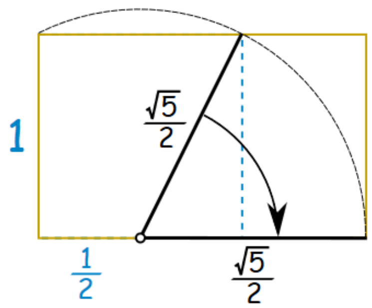
[8]

Yet another way to derive the golden ratio value is with the help of geometry and the Pythagoras theorem. Consider a square with each side equal to 1 centimetre. Draw a line from the top right corner of the square to the midpoint of the base of the square. The length of this line can be found in centimetres using the Pythagoras theorem.

$$length\ of\ line = \sqrt{1^2 + 0.5^2} = \sqrt{1.25} = \frac{\sqrt{5}}{2}$$

Rotate the line clockwise until it coincides with the base of the square.

The length of the base of the extended rectangle is now equal to$\frac{1+\sqrt{5}}{2}$, the golden ratio. This extended rectangle, called the Golden rectangle, has the dimensions of 1 is to 1.618.



The golden ratio can also be expressed in terms of the tangent of an angle. If the diagonal of the extended rectangle is drawn, a right-angled triangle with base 1.618 cm and height 1 cm is obtained. If the angle θ is taken such that the opposite side is equal to 1.618 cm and the adjacent side is equal to 1 cm, we have that:

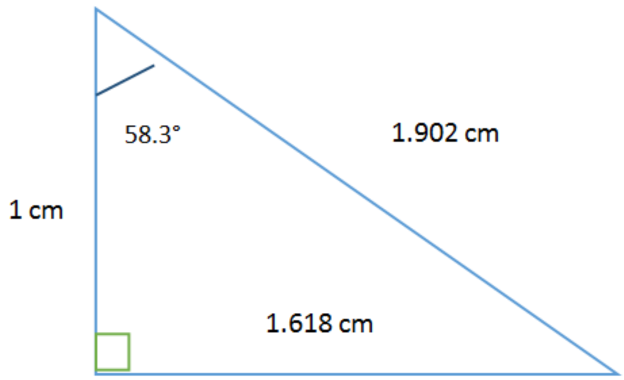
$$\tan\theta = \frac{1.618}{1}$$

$$\theta \approx \tan^{-1} 1.618$$

$$\theta \approx 58.3°$$

$$\therefore \varphi = \tan 58.3°$$

The golden ratio can be expressed in terms of the tangent of an angle. Similarly, the golden ratio can also be expressed in terms of the sine, cosine, secant, cosecant and cotangent of an angle.



$$hypotenuse = \sqrt{1^2 + 1.618^2} = \sqrt{3.617924}$$

$$\approx 1.902\ cm$$

$$\sin 58.3° = \frac{1.618}{1.902}$$

$$\varphi = 1.902 \sin 58.3°$$

$$\therefore \varphi = \frac{1.902}{\csc 58.3°}$$

$$\cos 31.7° = \frac{1.618}{1.902}$$

$$\varphi = 1.902 \cos 31.7°$$

$$\therefore \varphi = \frac{1.902}{\sec 31.7°}$$

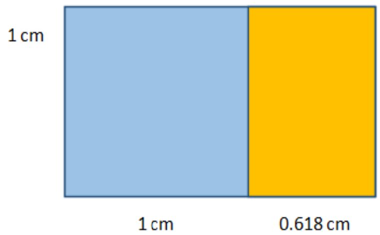
$$\varphi = \tan 58.3°$$

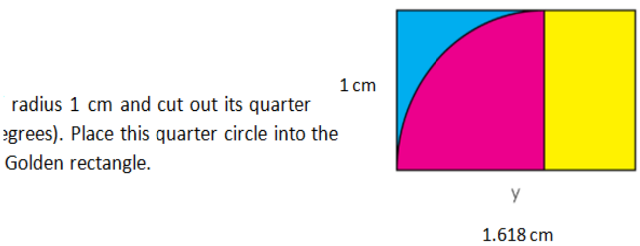
$$\therefore \varphi = \frac{1}{\cot 58.3°}$$

[8]Both images from "Golden Ratio - Math is Fun." https://www.mathsisfun.com/numbers/golden-ratio.html. Accessed 1 Aug. 2018.

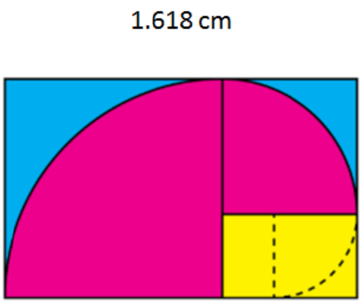
THE DIFFERENT GOLDEN SHAPES

An even more interesting and useful shape can be constructed using multiple Golden rectangles. First take a simple golden rectangle and divide it into a 1 cm by 1 cm square and a 1 cm by 0.618 cm rectangle:



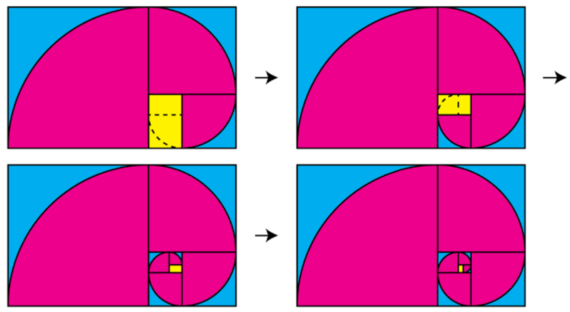


Now, take a circle of radius 1 cm and cut out its quarter part (section of 90 degrees).[9]

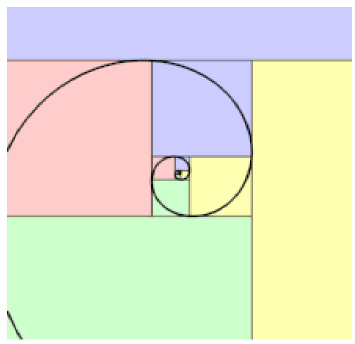


Now, if another golden rectangle with a square section of side 1.618 cm is taken, the above shape can be inserted into the other rectangular section. The resulting figure will look this:

If this pattern is continuously created within the small yellow golden rectangles, a characteristic curve is obtained that looks like a spiral.



If the outer curve on the last diagram is isolated, the result is a spiral. This is called the Golden spiral, and can be seen more clearly from the diagram alongside:



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The Golden spiral is a special type of logarithmic spiral. Logarithmic spirals are such that the angle created when a line goes from the centre of the spiral to the tangent of the spiral is always constant. The unique thing about the Golden spiral is that for every 90 degree turn of the spiral, the point at which you are tracing will travel a distance equal to the golden ratio from the centre of the spiral. That is, one will be about 1.618 cm further away from the centre of the spiral after a 90 degree turn relative to the centre.

Logarithmic spirals are generally modelled using the following equation:

$$r(\theta) = ae^{\theta \cot b}$$

This is a polar equation, with **r** representing the distance from the centre of the spiral and θ representing the angle relative to the centre. One can derive an equation for the Golden ratio for the case where **a** is equal to 1:

$$r(\theta) = e^{\theta \cot b}$$

$$\varphi = e^{\frac{\pi}{2} \cot b}$$

$$\frac{\pi}{2} \cot b = \ln \varphi$$

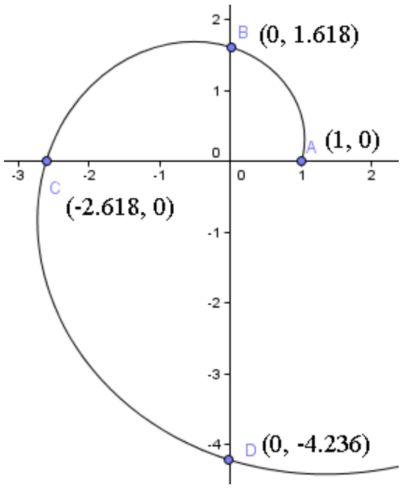
[9] All three images from "Golden Ratio - Math is Fun." https://www.mathsisfun.com/numbers/golden-ratio.html. Accessed 24 Dec. 2018.

[10] Image from "Golden spiral - Wikipedia." https://en.wikipedia.org/wiki/Golden_spiral. Accessed 21 Jan. 2019.

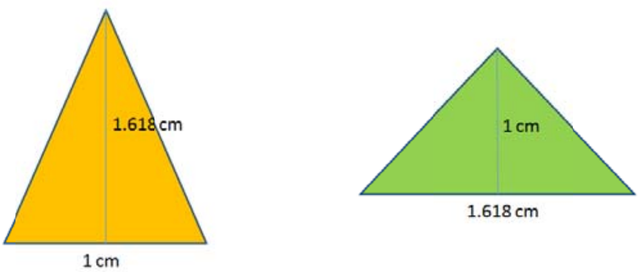
$$\cot b = \frac{\ln \varphi}{\frac{\pi}{2}} \approx 0.3063$$

$$\therefore r(\theta) = e^{0.30630\theta}$$

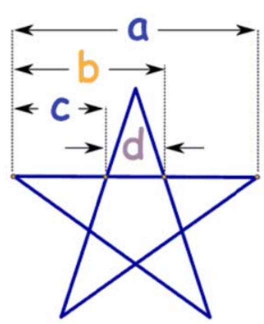
If the above Golden Equation is graphed on the polar plane, a spiral is obtained that almost perfectly models the Golden spiral.



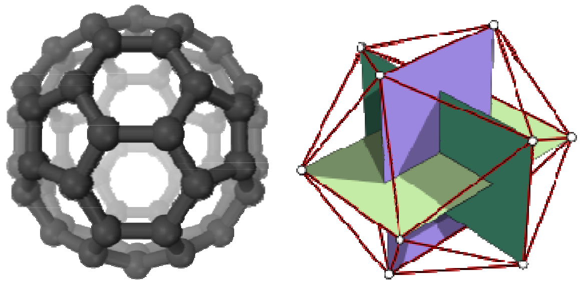
Almost any other shape can be classified as golden. For example, the Golden triangle has the height as 1.618 cm and base as 1 cm or the other way round.

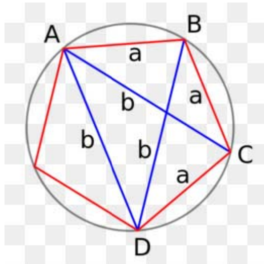


The figure alongside is a regular Golden pentagram. It is the typical symbol one uses to represent a star. This figure also serves as a holy symbol. The golden ratio can be observed in several parts of the pentagram.



For example, the ratios of **a** to**b**, **b** to **c**, and **c** to **d** are all numerically equal to the golden ratio.





The three figures above are a Bucky ball (Buckminster Fullerene), an icosahedrons, and a dodecagon respectively. Golden ratio patterns can be seen in these complex 3-D structures as well.

Buckminster Fullerene Balls, popularised by American architect Buckminster Fuller, have a shape that's really close to a sphere or a soccer ball. These Bucky balls are essentially carbon allotropes[11] containing 60 carbon atoms bonded together by strong covalent bonds[12]. They also contain 12 phi-based pentagonal faces and 20 phi-based hexagonal faces. If these Bucky balls are placed in the three-dimensional Cartesian plane with the centres at the origin, all 60 points of the Bucky balls can be represented in terms of phi.

For example, six of the points can be represented using the following coordinates:

(0, +1, +3Φ), (0,-1,-3Φ), (+1, +3Φ, 0), (-1,-3Φ, 0), (+3Φ, 0, +1), (-3Φ, 0,-1)

Icosahedrons and dodecagons also have the same properties as the Bucky balls mentioned above.

CONCLUSION

The Golden ratio, also called the "Divine Proportion" or "God's number", is omnipresent, occurs in various fields, and

[11]Each of two or more different physical forms in which an element can exist. – ("Allotropy - Wikipedia." https://en.wikipedia.org/wiki/Allotropy. Accessed 19 May. 2019.)

[12]A covalent bond, also called a molecular bond, is a chemical bond that involves the sharing of electron pairs between atoms. – ("Covalent bond - Wikipedia." https://en.wikipedia.org/wiki/Covalent_bond. Accessed 19 May. 2019.)

is pervasive in our day-to-day lives and the environment. However, spirituality sparks the question of whether god himself bestowed such a unique number upon the world or humanity has managed to discover the number that may have answers to long-standing questions, such as the shape of the universe. It is also sometimes believed that we might be perceiving patterns that just don’t exist. We may have conveniently attributed applications to the Golden ratio to show its omnipresence. Whatever the case, the Golden ratio has indeed been a highly controversial and hotly debated topic in the mathematics community of today and may as well play a huge role in the future.

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